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Semicompactness in L-fuzzy topological spaces

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Abstract. The aim of this paper is to introduce the notion of L-fuzzy semicompactness in L-fuzzy topological spaces, which is a generalization of semicompactness in L-topological spaces. The union of two L-fuzzy semicompact L-sets is L-fuzzy semicompact. The intersection of an L-fuzzy semicompact L-set G and an L-set H with $\mathcal{T}_s^*(H) = \top$ is L-fuzzy semicompact. The L-fuzzy irresolute image of an L-fuzzy semicompact L-set is L-fuzzy semicompact. The L-fuzzy semicontinuous image of an L-fuzzy semicompact L-set is L-fuzzy compact. The L-fuzzy strong irresolute image of an L-fuzzy compact L-set is L-fuzzy semicompact.

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1. Introduction

Lowen's fuzzy compactness [6, 7] is generalized into L-topological spaces by means of open L-sets and their inequality in [10]. Following the idea of [10], the notion of semicompactness [1] was also generalized into L-topological spaces [9]. Then a natural problem is: Can the notion of semicompactness be defined in an L-fuzzy topological space? In this paper, our aim is to introduce the notion of semicompactness in L-fuzzy topological spaces by means of L-fuzzy semiopen operators [11].

2. Preliminaries

Throughout this paper $(L, \bigvee, \bigwedge, ')$ is a completely distributive De Morgan algebra, X is a nonempty set and L^X is the set of all L-fuzzy sets on X. The smallest element and the largest element in L are denoted respectively by $\bot$ and $\top$. The smallest element and the largest element in L^X are denoted respectively by $\underline{\bot}$ and $\underline{\top}$. An L-fuzzy set is briefly written as an L-set. We often do not distinguish a crisp subset A from its characteristic function χ_A. The set of nonunit prime elements in L is denoted by $P(L)$. The set of nonzero co-prime elements in L is denoted by $M(L)$.

The binary relation $\prec$ in L is defined as follows: for $a, b \in L$, $a \prec b$ if and only if for every subset $D \subseteq L$, $b \leq \sup D$ always implies the existence of $d \in D$ with $a \leq d$ [2]. In a completely distributive DeMorgan algebra L, each member b is a sup of $\{a \in L \mid a \prec b\}$. In the sense of [5, 14], $\{a \in L \mid a \prec b\}$ is the greatest minimal family of b, denoted by $\beta(b)$, and $\beta^*(b) = \beta(b) \cap M(L)$. Moreover for $b \in L$, define $\alpha(b) = \{a \in L \mid a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \cap P(L)$.

Definition 2.1 ([4, 13])**.** An L-fuzzy topology on a set X is a map $\mathcal{T} : L^X \to L$ such that

(1) $\mathcal{T}(\underline{\top}) = \mathcal{T}(\underline{\perp}) = \top$;
(2) $\forall U, V \in L^X$, $\mathcal{T}(U \wedge V) \geq \mathcal{T}(U) \wedge \mathcal{T}(V)$;
(3) $\forall U_j \in L^X$, $j \in J$, $\mathcal{T}(\bigvee_{j\in J} U_j) \geq \bigwedge_{j\in J} \mathcal{T}(U_j)$.

$\mathcal{T}(U)$ can be interpreted as the degree to which U is an open set. $\mathcal{T}^*(U) = \mathcal{T}(U')$ will be called the degree of closedness of U. The pair $(X, \mathcal{T})$ is called an L-fuzzy topological space.

A mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is said to be L-fuzzy continuous if $\mathcal{T}_1(f_L^{\leftarrow}(B)) \geq \mathcal{T}_2(B)$ holds for all $B \in L^Y$, where $f_L^{\leftarrow}$ is defined by $f_L^{\leftarrow}(B)(x) = B(f(x))$ (see [8]).

Definition 2.2 ([10])**.** Let $a \in L\backslash\{\top\}$ and $G \in L^X$. A subfamily $\mathcal{U}$ in L^X is said to be

(1) an a-shading of G if for any $x \in X$, it follows that $G'(x) \vee \bigvee_{A\in\mathcal{U}} A(x) \not\leq a$.

(2) a strong a-shading of G if $\bigwedge_{x\in X} \left(G'(x) \vee \bigvee_{A\in\mathcal{U}} A(x) \right) \not\leq a$.

Definition 2.3 ([10])**.** Let $a \in L\backslash\{\perp\}$ and $G \in L^X$. A subfamily $\mathcal{P}$ in L^X is said to be

(1) an a-remote family of G if for any $x \in X$, it follows that $G(X) \wedge \bigwedge_{B\in\mathcal{P}} B(x) \not\geq a$.

(2) a strong a-remote family of G if $\bigvee_{x\in X} \left(G(x) \wedge \bigwedge_{B\in\mathcal{P}} B(x) \right) \not\geq a$.

Definition 2.4 ([10])**.** Let $a \in L\backslash\{\perp\}$ and $G \in L^X$. A subfamily $\mathcal{U}$ in L^X is called

(1) a β_a-cover of G if for any $x \in X$, it follows that $a \in \beta \left(G'(x) \vee \bigvee_{A\in\mathcal{U}} A(x) \right)$.

(2) a strong β_a-cover of G if for any $x \in X$, it follows that

$$a \in \beta \left(\bigwedge_{x\in X} \left(G'(x) \vee \bigvee_{A\in\mathcal{U}} A(x) \right) \right).$$

(3) a Q_a-cover of G if $a \leq \bigwedge_{x\in X} \left(G'(x) \vee \bigvee_{A\in\mathcal{U}} A(x) \right)$.

Definition 2.5 ([11])**.** Let $\mathcal{T}$ be an L-fuzzy topology on X. For any $A \in L^X$, define a mapping $\mathcal{T}_s : L^X \to L$ by

$$\mathcal{T}_s(A) = \bigvee_{B\leq A} \left\{ \mathcal{T}(B) \wedge \bigwedge_{x_\lambda \prec A} \bigwedge_{x_\lambda \not\leq D \geq B} (\mathcal{T}(D'))' \right\}.$$

Then $\mathcal{T}_s$ is called the L-fuzzy semiopen operator induced by $\mathcal{T}$, where $\mathcal{T}_s(A)$ can be regarded as the degree to which A is semiopen and $\mathcal{T}_s^*(B) = \mathcal{T}_s(B')$ can be regarded as the degree to which B is semiclosed.

Theorem 2.6 ([11])**.** *Let $\mathcal{T}$ be an L-fuzzy topology on X and let $\mathcal{T}_s$ be the L-fuzzy semiopen operator induced by $\mathcal{T}$. Then $\mathcal{T}(A) \leq \mathcal{T}_s(A)$ for any $A \in L^X$.*

Definition 2.7 ([11])**.** A mapping $f : X \to Y$ between two L-fuzzy topological spaces $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ is called

(1) semicontinuous if $\mathcal{T}_2(U) \leq (\mathcal{T}_1)_s(f_L^{\leftarrow}(U))$ holds for any $U \in L^Y$;
(2) irresolute if $(\mathcal{T}_2)_s(U) \leq (\mathcal{T}_1)_s(f_L^{\leftarrow}(U))$ holds for any $U \in L^Y$.

Theorem 2.8 ([11])**.** *If $f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is continuous with respect to L-fuzzy topologies $\mathcal{T}_1$ and $\mathcal{T}_2$, then f is also semicontinuous.*

Theorem 2.9 ([11])**.** *If $f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is irresolute, then f is semicontinuous.*

Definition 2.10 ([12])**.** Let $(X, \mathcal{T})$ be an L-fuzzy topological space. $G \in L^X$ is said to be L-fuzzy compact if for every family $\mathcal{U} \subseteq L^X$, it follows that

$$\bigwedge_{F\in\mathcal{U}} \mathcal{T}(F) \wedge \left(\bigwedge_{x\in X} \left(G'(x) \vee \bigvee_{F\in\mathcal{U}} F(x) \right) \right) \leq \bigvee_{\mathcal{V}\in 2^{(\mathcal{U})}} \bigwedge_{x\in X} \left(G'(x) \vee \bigvee_{F\in\mathcal{V}} F(x) \right).$$

3. Definition and characterizations of L-fuzzy semicompactness

Definition 3.1. Let $(X, \mathcal{T})$ be an L-fuzzy topological space. $G \in L^X$ is said to be L-fuzzy semicompact if for every family $\mathcal{U} \subseteq L^X$, it follows that

$$\bigwedge_{A\in\mathcal{U}} \mathcal{T}_s(A) \wedge \left(\bigwedge_{x\in X} \left(G'(x) \vee \bigvee_{A\in\mathcal{U}} A(x) \right) \right) \leq \bigvee_{\mathcal{V}\in 2^{(\mathcal{U})}} \bigwedge_{x\in X} \left(G'(x) \vee \bigvee_{A\in\mathcal{V}} A(x) \right).$$

By Theorem 2.6, Definition 2.10 and Definition 3.1 we can obtain the following result.

Theorem 3.2. *L-fuzzy semicompactness implies L-fuzzy compactness.*

Let $(X, \mathcal{T})$ be an L-topological space. Let $\chi_{\mathcal{T}} : L^X \to L$

$$\chi_{\mathcal{T}}(A) = \begin{cases} 1, & A \in \mathcal{T}, \\ 0, & A \notin \mathcal{T}. \end{cases}$$

Obviously, $(X, \chi_{\mathcal{T}})$ is a special L-fuzzy topological spaces. So we can easily prove the following theorem.

Theorem 3.3. *Let $(X, \mathcal{T})$ be an L-topological space and $G \in L^X$. G is L-fuzzy semicompact in $(X, \chi_{\mathcal{T}})$ if and only if G is fuzzy semicompact in $(X, \mathcal{T})$.*

From Definition 3.1 we easily obtain the following theorem by simply using quasi-complement $'$.

Theorem 3.4. *Let $(X, \mathcal{T})$ be an L-fuzzy topological space. $G \in L^X$ is L-fuzzy semicompact if and only if for every family $\mathcal{P} \subseteq L^X$ it follows that*

$$\bigvee_{F\in\mathcal{P}} (\mathcal{T}_s^*(F))' \vee \left(\bigvee_{x\in X} \left(G(x) \wedge \bigwedge_{F\in\mathcal{P}} F(x) \right) \right) \geq \bigwedge_{\mathcal{H}\in 2^{(\mathcal{P})}} \bigvee_{x\in X} \left(G(x) \wedge \bigwedge_{F\in\mathcal{H}} F(x) \right).$$

By Definition 3.1 and Theorem 3.4 we immediately obtain the following two theorems.

Theorem 3.5. *Let $(X, \mathcal{T})$ be an L-fuzzy topological space and $G \in L^X$. Then the following conditions are equivalent to each other.*

(1) *G is L-fuzzy semicompact.*
(2) *For any $a \in M(L)$, each strong a-remote family $\mathcal{P}$ of G with $\bigwedge_{F\in\mathcal{P}} \mathcal{T}_s^*(F) \not\leq a'$ has a finite subfamily $\mathcal{H}$ which is a strong a-remote family of G.*
(3) *For any $a \in M(L)$, each strong a-remote family $\mathcal{P}$ of G with $\bigwedge_{F\in\mathcal{P}} \mathcal{T}_s^*(F) \not\leq a'$ has a finite subfamily $\mathcal{H}$ which is an a-remote family of G.*
(4) *For any $a \in M(L)$, and any strong a-remote family $\mathcal{P}$ of G with $\bigwedge_{F\in\mathcal{P}} \mathcal{T}_s^*(F) \not\leq a'$, there exists a finite subfamily $\mathcal{H}$ of $\mathcal{P}$ and $b \in \beta^*(a)$ such that $\mathcal{H}$ is a strong b-remote family of G.*
(5) *For any $a \in M(L)$, and any strong a-remote family $\mathcal{P}$ of G with $\bigwedge_{F\in\mathcal{P}} \mathcal{T}_s^*(F) \not\leq a'$, there exists a finite subfamily $\mathcal{H}$ of $\mathcal{P}$ and $b \in \beta^*(a)$ such that $\mathcal{H}$ is a b-remote family of G.*
(6) *For any $a \in P(L)$, each strong a-shading $\mathcal{U}$ of G with $\bigwedge_{F\in\mathcal{U}} \mathcal{T}_s(F) \not\leq a$ has a finite subfamily $\mathcal{V}$ which is a strong a-shading of G.*
(7) *For any $a \in P(L)$, each strong a-shading $\mathcal{U}$ of G with $\bigwedge_{F\in\mathcal{U}} \mathcal{T}_s(F) \not\leq a$ has a finite subfamily $\mathcal{V}$ which is an a-shading of G.*
(8) *For any $a \in P(L)$ and any strong a-shading $\mathcal{U}$ of G with $\bigwedge_{F\in\mathcal{U}} \mathcal{T}_s(F) \not\leq a$, there exists a finite subfamily $\mathcal{V}$ of $\mathcal{U}$ and $b \in \alpha^*(a)$ such that $\mathcal{V}$ is a strong b-shading of G.*
(9) *For any $a \in P(L)$ and any strong a-shading $\mathcal{U}$ of G with $\bigwedge_{F\in\mathcal{U}} \mathcal{T}_s(F) \not\leq a$, there exists a finite subfamily $\mathcal{V}$ of $\mathcal{U}$ and $b \in \alpha^*(a)$ such that $\mathcal{V}$ is a b-shading of G.*
(10) *For any $a \in M(L)$ and any $b \in \beta^*(a)$, each Q_a-cover $\mathcal{U}$ of G with $\mathcal{T}_s(F) \geq a$ $(\forall F \in \mathcal{U})$ has a finite subfamily $\mathcal{V}$ which is a Q_b-cover of G.*
(11) *For any $a \in M(L)$ and any $b \in \beta^*(a)$, each Q_a-cover $\mathcal{U}$ of G with $\mathcal{T}_s(F) \geq a$ $(\forall F \in \mathcal{U})$ has a finite subfamily $\mathcal{V}$ which is a strong β_b-cover of G.*
(12) *For any $a \in M(L)$ and any $b \in \beta^*(a)$, each Q_a-cover $\mathcal{U}$ of G with $\mathcal{T}_s(F) \geq a$ $(\forall F \in \mathcal{U})$ has a finite subfamily $\mathcal{V}$ which is a β_b-cover of G.*

Theorem 3.6. *Let $(X, \mathcal{T})$ be an L-fuzzy topological space and $G \in L^X$. If $\beta(c \wedge d) = \beta(c) \cap \beta(d)$ $(\forall c, d \in L)$, then the following conditions are equivalent to each other.*

(1) *G is L-fuzzy semicompact.*
(2) *For any $a \in M(L)$, each strong β_a-cover $\mathcal{U}$ of G with $a \in \beta\left(\bigwedge_{F\in\mathcal{U}} \mathcal{T}(F)\right)$ has a finite subfamily $\mathcal{V}$ which is a strong β_a-cover of G.*
(3) *For any $a \in M(L)$, each strong β_a-cover $\mathcal{U}$ of G with $a \in \beta\left(\bigwedge_{F\in\mathcal{U}} \mathcal{T}(F)\right)$ has a finite subfamily $\mathcal{V}$ which is a β_a-cover of G.*

(4) *For any* $a \in M(L)$ *and any strong* β_a*-cover* $\mathcal{U}$ *of* G *with* $a \in \beta\left(\bigwedge_{F\in\mathcal{U}} \mathcal{T}(F)\right)$, *there exists a finite subfamily* $\mathcal{V}$ *of* $\mathcal{U}$ *and* $b \in M(L)$ *with* $a \in \beta^*(b)$ *such that* $\mathcal{V}$ *is a strong* β_b*-cover of* G.

(5) *For any* $a \in M(L)$ *and any strong* β_a*-cover* $\mathcal{U}$ *of* G *with* $a \in \beta\left(\bigwedge_{F\in\mathcal{U}} \mathcal{T}(F)\right)$, *there exists a finite subfamily* $\mathcal{V}$ *of* $\mathcal{U}$ *and* $b \in M(L)$ *with* $a \in \beta^*(b)$ *such that* $\mathcal{V}$ *is a* β_b*-cover of* G.

4. Properties of L-fuzzy semicompactness

Theorem 4.1. *Let* $(X, \mathcal{T})$ *be an L-fuzzy topological space and* $G \in L^X$*. If G is L-fuzzy semicompact, then for each* $H \in L^X$ *with* $\mathcal{T}_s^*(H) = \top$, $G \wedge H$ *is L-fuzzy semicompact.*

Proof. The L-fuzzy semicompactness of $G \wedge H$ can be proved from the following fact.

$$\begin{aligned}
&\bigvee_{F\in\mathcal{P}} (\mathcal{T}_s^*(F))' \vee \left(\bigvee_{x\in X}\left((G\wedge H)(x) \wedge \bigwedge_{F\in\mathcal{P}} F(x)\right)\right)\\
&= \bigvee_{F\in\mathcal{P}\bigcup\{H\}} (\mathcal{T}_s^*(F))' \vee \left(\bigvee_{x\in X}\left(G(x) \wedge \bigwedge_{F\in\mathcal{P}\bigcup\{H\}} F(x)\right)\right)\\
&\geq \bigwedge_{\mathcal{F}\in 2^{(\mathcal{P}\cup\{H\})}} \bigvee_{x\in X}\left(G(x) \wedge \bigwedge_{F\in\mathcal{F}} F(x)\right)\\
&= \bigwedge_{\mathcal{F}\in 2^{(\mathcal{P})}} \bigvee_{x\in X}\left(G(x) \wedge H(x) \wedge \bigwedge_{F\in\mathcal{F}} F(x)\right).
\end{aligned}$$

□

Theorem 4.2. *Let* $(X, \mathcal{T})$ *be an L-fuzzy topological space and* $G, H \in L^X$*. If both G and H are L-fuzzy semicompact, then so is* $G \vee H$

Proof. This can be proved from the following fact.

$$\begin{aligned}
&\bigvee_{F\in\mathcal{P}} (\mathcal{T}_s^*(F))' \vee \left(\bigvee_{x\in X}\left((G\vee H)(x) \wedge \bigwedge_{F\in\mathcal{P}} F(x)\right)\right)\\
&= \bigvee_{F\in\mathcal{P}} (\mathcal{T}_s^*(F))' \vee \left(\bigvee_{x\in X}\left(G(x) \wedge \bigwedge_{F\in\mathcal{P}} F(x)\right)\right) \vee \left(\bigvee_{x\in X}\left(H(x) \wedge \bigwedge_{F\in\mathcal{P}} F(x)\right)\right)\\
&\geq \bigwedge_{\mathcal{Q}_1\in 2^{(\mathcal{P})}} \bigvee_{x\in X}\left(G(x) \wedge \bigwedge_{F\in\mathcal{Q}_1} F(x)\right) \vee \bigwedge_{\mathcal{Q}_2\in 2^{(\mathcal{P})}} \bigvee_{x\in X}\left(H(x) \wedge \bigwedge_{F\in\mathcal{Q}_2} F(x)\right)\\
&\geq \bigwedge_{\mathcal{Q}\in 2^{(\mathcal{P})}} \bigvee_{x\in X}\left((G\vee H)(x) \wedge \bigwedge_{F\in\mathcal{Q}} F(x)\right).
\end{aligned}$$

□

Theorem 4.3. *Let $(X,\mathcal{T}_1)$, $(Y,\mathcal{T}_2)$ be two L-fuzzy topological spaces, and $f:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ be an L-fuzzy irresolute mapping. If $G\in L^X$ is L-fuzzy semicompact in $(X,\mathcal{T}_1)$, then so is $f_L^{\to}(G)$ in $(Y,\mathcal{T}_2)$.*

Proof. This can be proved from the following fact.

$$\begin{aligned}
&\bigvee_{F\in\mathcal{P}}((\mathcal{T}_2)_s^*(F))'\vee\left(\bigvee_{y\in Y}\left(f_L^{\to}(G)(y)\wedge\bigwedge_{F\in\mathcal{P}}F(y)\right)\right)\\
&\geq\bigvee_{F\in\mathcal{P}}((\mathcal{T}_1)_s^*(f_L^{\leftarrow}(F)))'\vee\left(\bigvee_{x\in X}\left(G(x)\wedge\bigwedge_{F\in\mathcal{P}}f_L^{\leftarrow}(F)(x)\right)\right)\\
&\geq\bigwedge_{\mathcal{F}\in 2^{(\mathcal{P})}}\bigvee_{x\in X}\left(G(x)\wedge\bigwedge_{F\in\mathcal{F}}f_L^{\leftarrow}(F)(x)\right)\\
&\geq\bigwedge_{\mathcal{F}\in 2^{(\mathcal{P})}}\bigvee_{y\in Y}\left(f_L^{\to}(G)(y)\wedge\bigwedge_{F\in\mathcal{F}}F(y)\right).
\end{aligned}$$

□

Analogously we can obtain the following result.

Theorem 4.4. *Let $(X,\mathcal{T}_1)$, $(Y,\mathcal{T}_2)$ be two L-fuzzy topological spaces, and $f:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ be an L-fuzzy semicontinuous mapping. If $G\in L^X$ is L-fuzzy semicompact in $(X,\mathcal{T}_1)$, then $f_L^{\to}(G)$ is L-fuzzy compact in $(Y,\mathcal{T}_2)$.*

Definition 4.5. Let $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ be two L-fuzzy topological spaces. A mapping $f:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is called strongly irresolute if $(\mathcal{T}_2)_s(U)\leq\mathcal{T}_1(f_L^{\leftarrow}(U))$ holds for any $U\in L^Y$.

It is obvious that a strongly irresolute mapping is irresolute. Analogously we have the following result.

Theorem 4.6. *Let $(X,\mathcal{T}_1)$, $(Y,\mathcal{T}_2)$ be two L-fuzzy topological spaces, and $f:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ be an L-fuzzy strong irresolute mapping. If $G\in L^X$ is L-fuzzy compact in $(X,\mathcal{T}_1)$, then $f_L^{\to}(G)$ is L-fuzzy semicompact in $(Y,\mathcal{T}_2)$.*

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